





An Automatic Finite-Sample Robustness Metric: Can Dropping a Little Data Change Conclusions?

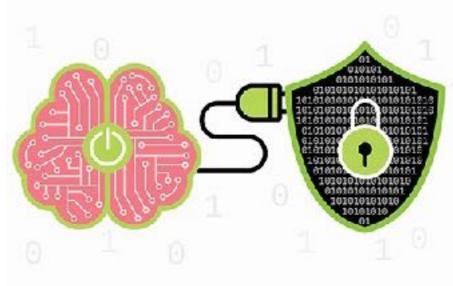
Tamara Broderick
Associate Professor,
MIT

With Ryan Giordano, Rachael Meager





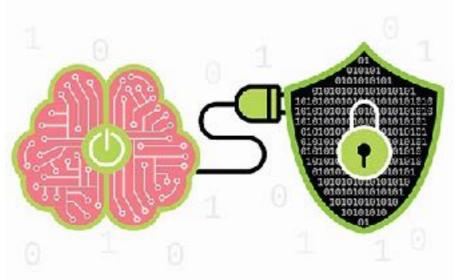






 More data & better computation → data analyses increasingly drive life-changing decisions

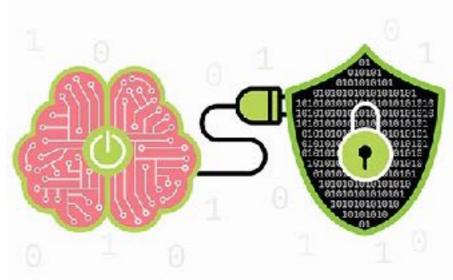






 One question: Would you be concerned if dropping a small fraction of data changed substantive conclusions?

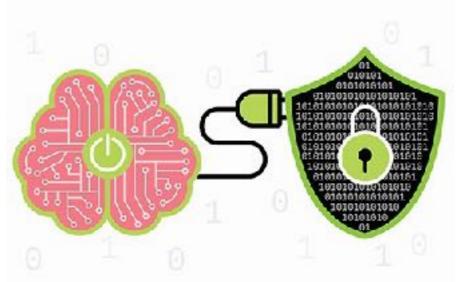






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- Challenge: Too expensive to check every data subset

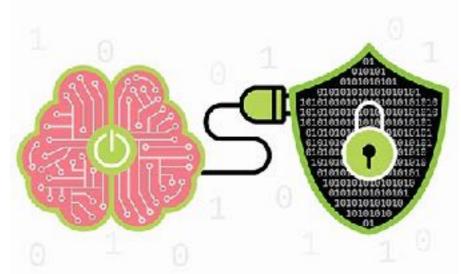






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- E.g. in a study of microcredit with ~16,500 data points, we find a single data point that drives the sign of the effect

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- Even if doesn't bother you, should be up front about it

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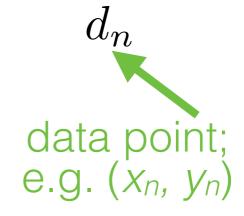
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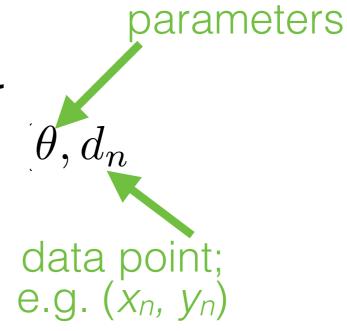
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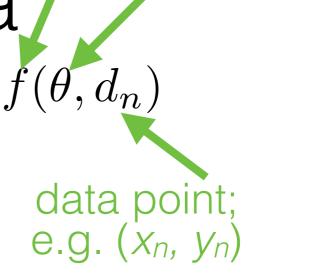
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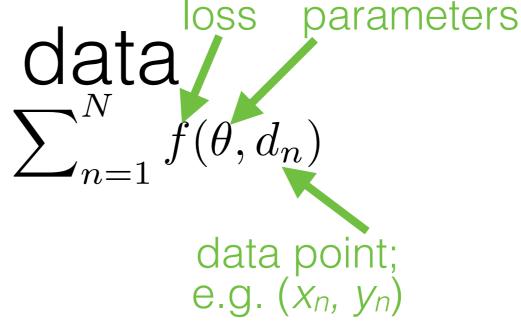
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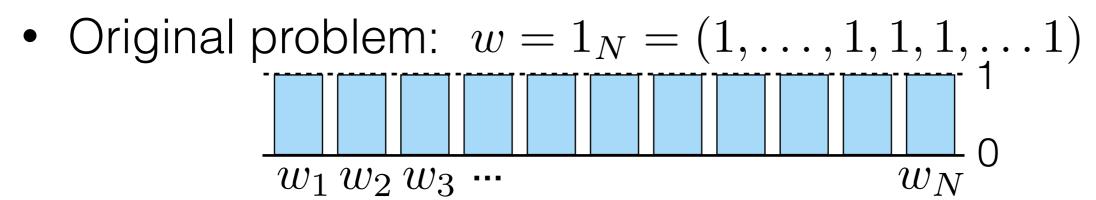
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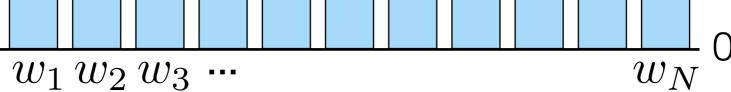
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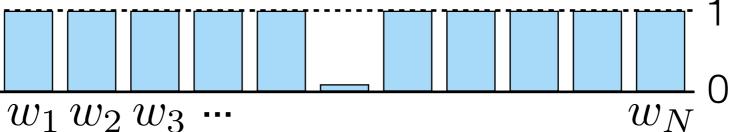
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parameters

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• Dropping a data point: $w = (1, \dots, 1, 0, 1, \dots, 1)$



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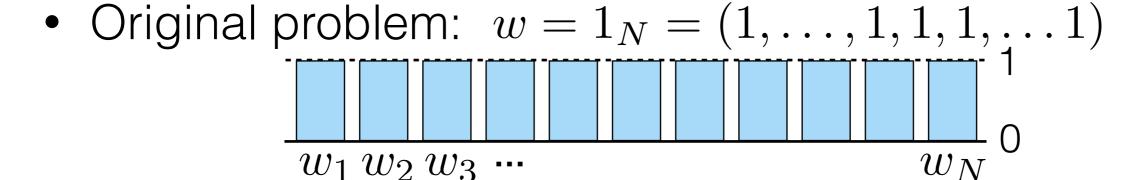
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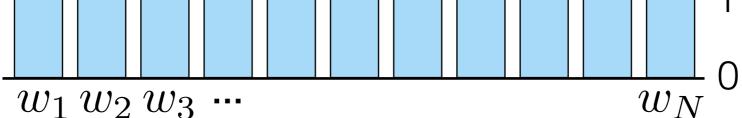
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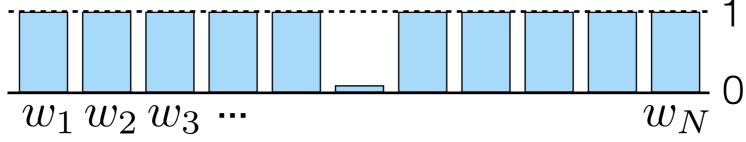
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Simulations from linear model with Gaussian noise

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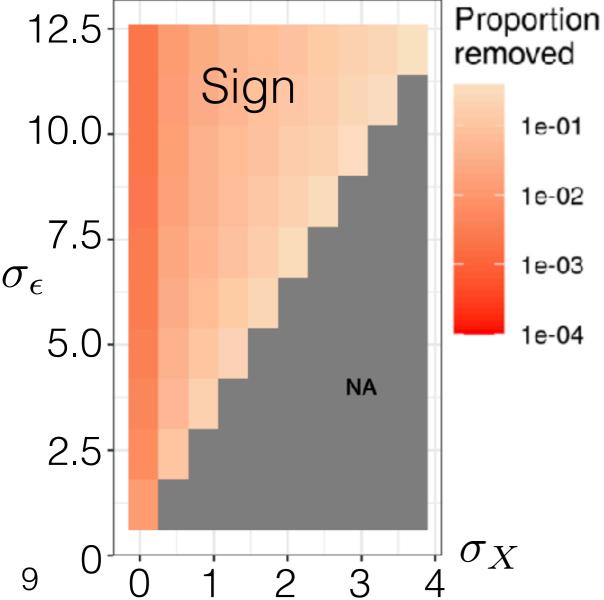
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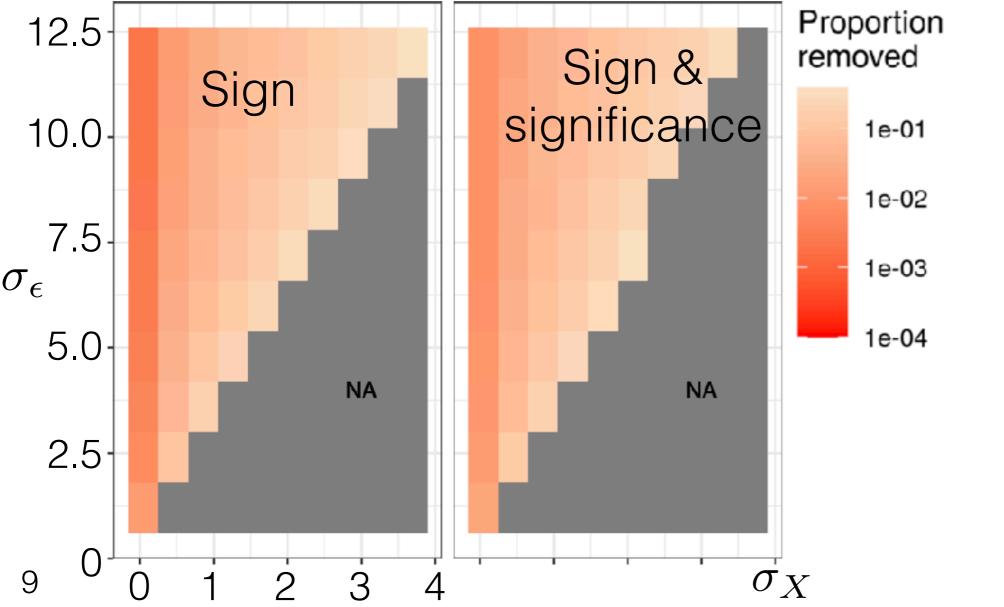
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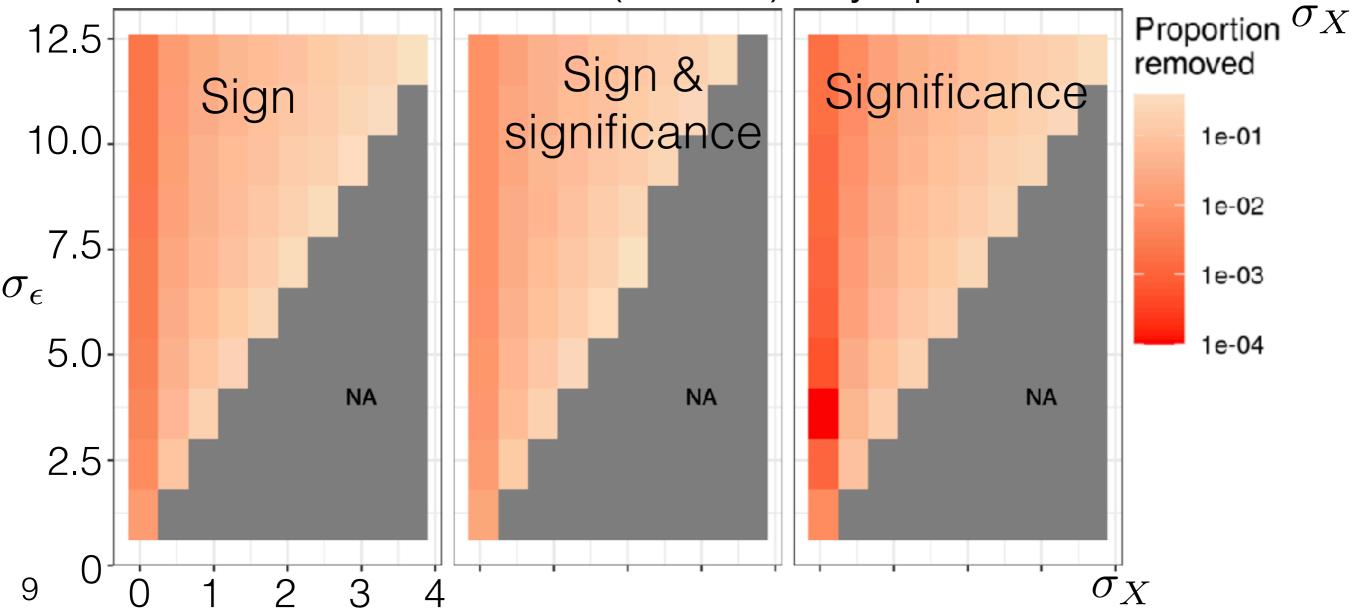
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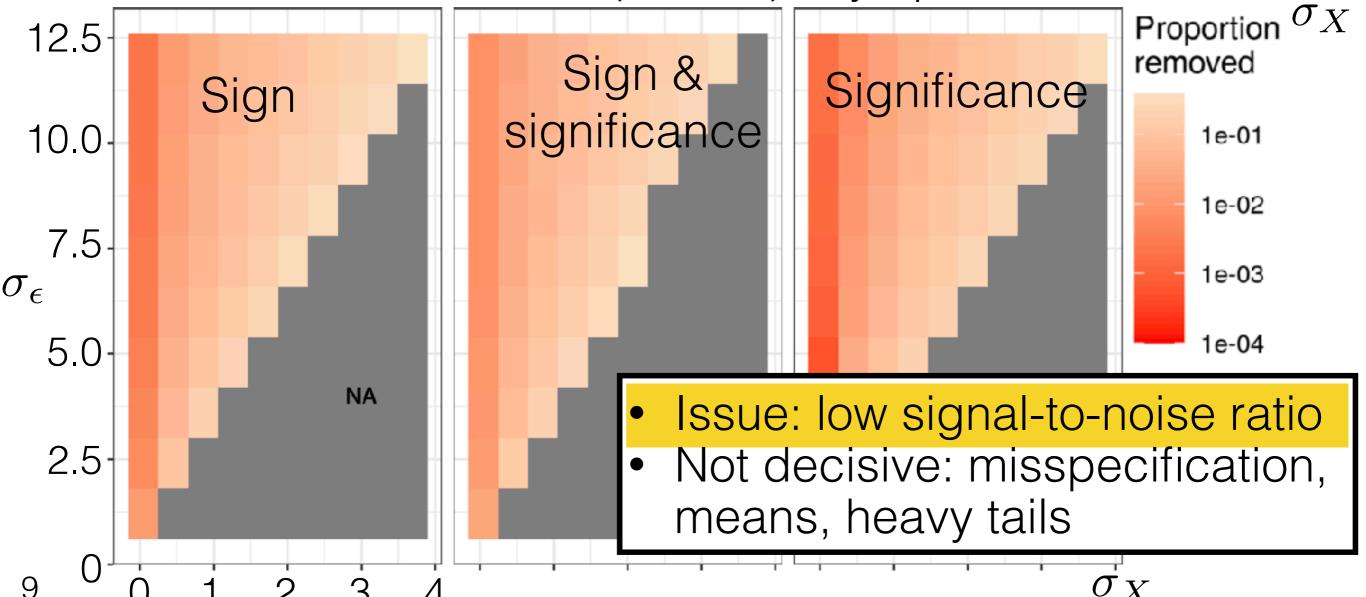
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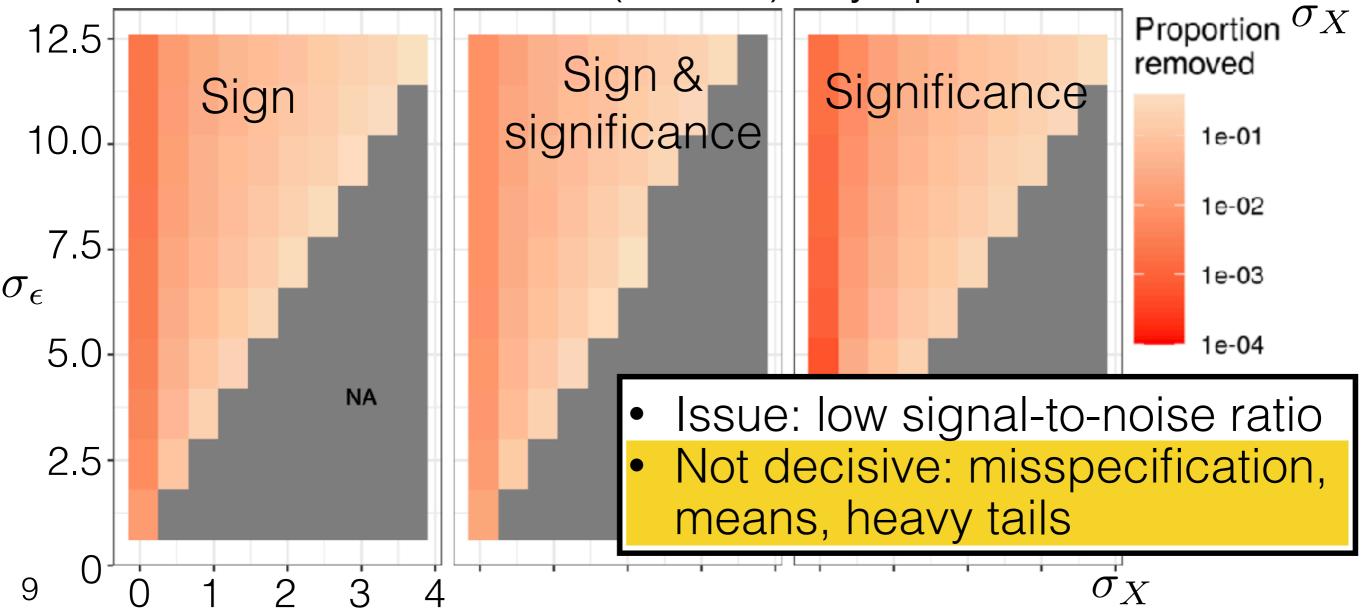
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 - Cf. the classical "infinitesimal jackknife" [Jaeckel 1972; Clarke 1983]

Try it out!

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```
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```

• Code, readme, and examples:

```
https://github.com/rgiordan/zaminfluence
```

Try it out on your data analysis and email us!

```
tbroderick@mit.edu,
rgiordan@mit.edu,
r.meager@lse.ac.uk
```

 Aside: "Transparency and Reproducibility in Artificial Intelligence," Nature Matters Arising, 2020.

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- See also:
 - R Giordano, T Broderick, MI Jordan. Linear Response Methods for Accurate Covariance Estimates from Mean Field Variational Bayes. NeurIPS 2015.
 - R Giordano, T Broderick, MI Jordan. Covariances, Robustness, and Variational Bayes. JMLR 2018.
 - R Giordano, W Stephenson, R Liu, MI Jordan, T Broderick. A Swiss Army infinitesimal jackknife. AISTATS 2019.